UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2021

CC8-MATHEMATICS

MULTIVARIATE CALCULUS

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer ***all*** questions: 2×5 = 10

(a) Examine, if the function $f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} &, \quad (x,\ y) \neq (0, 0) \\ 0 &, \quad x = y = 0 \end{cases}$

is continuous at the origin.

(b) For a conservative field **F**, prove that $\operatorname{curl}\mathbf{F} = \mathbf{0}$.

(c) Justify $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sqrt{x^2y^2+1}-1}{x^2+y^2} = 0$ using ε-δ definition.

(d) Let $\boldsymbol{F} = xy\,\boldsymbol{i} - z\boldsymbol{j} + x^2\boldsymbol{k}$ and Γ be a curve $x = t^2,\ y = 2t,\ z = t^3$ from $t = 0$ to $t = 1$. Evaluate the integral $\int \boldsymbol{F} \times d\boldsymbol{r}$ over the curve Γ.

(e) Evaluate $\iint\limits_R y\, e^{xy}\, dxdy$, where $R = \{(x,\ y) :\ 0 \le x \le a,\ \ 0 \le y \le b\}$.

GROUP-B

Answer *all* questions 10×3 = 30

2. (a) Verify Green's theorem in the plane for $\int [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ over the curve Γ, where Γ is the boundary of the region defined by $y = \sqrt{x},\ y = x^2$. 5

(b) Evaluate $\iint\limits_a \dfrac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}}\, dx\, dy$, the field of integration being R, the positive quadrant of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. 5

3. (a) Prove that for any vector function $\vec{f}$, curl $\vec{f} = \vec{\nabla}(\vec{\nabla} \cdot \vec{f}) - \nabla^2 \vec{f}$. 5

(b) Using Stokes theorem show that 5

$$\iint_S (y-z)\,dydz + (z-x)\,dzdx + (x-y)\,dxdy = a^3\pi,$$

where S is the portion of the surface $x^2 + y^2 - 2ax + az = 0$, $z \geq 0$.

4. (a) Find the work done by the force $\vec{F} = -y\vec{i} + x\vec{j} + z\vec{k}$ in moving a particle from (0, 0, 0) to (2, 4, 8) along a line segment and along the path $\vec{r} = t\vec{i} + t^2\vec{j} + t^3\vec{k}$. 5

(b) Compute the line integral $\int_\Gamma x^3 dx + 3zy^2 dy - x^2 y\, dz$, where Γ is the straight-line segment from (3, 2, 1) to (0, 0, 0). 5

GROUP-C

Answer *all* questions 5×2 = 10

5. Prove that $\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx$. Does the double integral $\iint \frac{x-y}{(x+y)^3} dxdy$ exist over $E = [0,1\,;\,0,1]$? Justify your answer. 5

6. If $x = u^2 v$, $y = v^2 u$, show that $2x^2 f_{xx} + 2y^2 f_{yy} + 5xy f_{xy} = uv f_{uv} - \frac{2}{3}(u f_u + v f_v)$. 5

GROUP-D

Answer *all* questions 5×2 = 10

7. Define the differentiability of a function $f(x, y)$ of two variables x, y at a point (a, b). Show that if $f(x, y)$ is differentiable at (a, b) then f is continuous at (a, b) and that the partial derivatives $f_x(a, b)$ and $f_y(a, b)$ exist. 5

8. Using Stoke's theorem prove that div curl $\vec{F} = 0$ and curl grad $\varphi = \vec{0}$. 5

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